$\boxed{2}$ `// relevant equations`

$$\left[\begin{array}{cc} \mathrm{Wn} = \sqrt{\frac{k}{m}} & \text{"natural frequency equation"} \\ \zeta = \frac{b}{2\sqrt{km}} & \text{"damping ratio equation"} \\ \mathrm{Wd} = \mathrm{Wn} \cdot \sqrt{-\zeta^2 + 1} & \text{"damped frequency equation"} \\ \zeta = \frac{\ln\left(\frac{x_0}{\mathrm{xn}}\right)}{\sqrt{4\pi^2 + \left(\frac{\ln\left(\frac{x_0}{\mathrm{xn}}\right)}{n}\right)^2 n}} & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = \frac{2\pi n}{-t_0 + \mathrm{tn}} & \text{"damped frequency from measurements"} \end{array}\right] \tag{1}$$

$\boxed{3}$ `// data from curve`

$$\left[\begin{array}{cc} t_0 = 4.15041 & \text{"start time"} \\ x_0 = 0.0702612 & \text{"starting position"} \\ \mathrm{tn} = 17.3807 & \text{"time at n cycles"} \\ \mathrm{xn} = 0.00203278 & \text{"position at n cycles"} \\ n = 3 & \text{"number of cycles"} \\ m = 1 & \text{"mass"} \end{array}\right] \tag{2}$$

$\boxed{4}$ `// substitute data into equations`

$$\left[\begin{array}{cc} \mathrm{Wn} = \sqrt{k} & \text{"natural frequency equation"} \\ \zeta = \frac{b}{2\sqrt{k}} & \text{"damping ratio equation"} \\ \mathrm{Wd} = \mathrm{Wn} \cdot \sqrt{-\zeta^2 + 1} & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{array}\right] \tag{3}$$

$\boxed{5}$ `// substitute calculated damping ratio and damped frequency into equations`

$$\left[\begin{array}{cc} \mathrm{Wn} = \sqrt{k} & \text{"natural frequency equation"} \\ \zeta = \frac{b}{2\sqrt{k}} & \text{"damping ratio equation"} \\ \mathrm{Wd} = \sqrt{k} \cdot 0.982791595626 & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{array}\right] \tag{4}$$

$\boxed{6}$ `// solve first equation for k`

$$\begin{bmatrix} k = \mathrm{Wn}^2 & \text{"natural frequency equation"} \\ \zeta = \frac{b}{2\sqrt{k}} & \text{"damping ratio equation"} \\ \mathrm{Wd} = \sqrt{k} \cdot 0.982791595626 & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{5}$$

$\boxed{7}$ `// substitute into equations`

$$\begin{bmatrix} k = \mathrm{Wn}^2 & \text{"natural frequency equation"} \\ \zeta = \frac{b}{2 \cdot \mathrm{Wn}} & \text{"damping ratio equation"} \\ \mathrm{Wd} = 0.982791595626 \cdot \mathrm{Wn} & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{6}$$

$\boxed{8}$ `// solve second equation for b`

$$\begin{bmatrix} k = \mathrm{Wn}^2 & \text{"natural frequency equation"} \\ b = 2\zeta \cdot \mathrm{Wn} & \text{"damping ratio equation"} \\ \mathrm{Wd} = 0.982791595626 \cdot \mathrm{Wn} & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{7}$$

$\boxed{9}$ `// substitute into equations`

$$\begin{bmatrix} k = \mathrm{Wn}^2 & \text{"natural frequency equation"} \\ b = 0.369435675413 \cdot \mathrm{Wn} & \text{"damping ratio equation"} \\ \mathrm{Wd} = 0.982791595626 \cdot \mathrm{Wn} & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{8}$$

$\boxed{10}$ `// solve third equation for natural frequency`

$$\begin{bmatrix} k = \mathrm{Wn}^2 & \text{"natural frequency equation"} \\ b = 0.369435675413 \cdot \mathrm{Wn} & \text{"damping ratio equation"} \\ \mathrm{Wn} = 1.01750971869 \cdot \mathrm{Wd} & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{9}$$

$\boxed{11}$ `// substitute into equations`

$$\begin{bmatrix} k = (1.01750971869 \cdot \mathrm{Wd})^2 & \text{"natural frequency equation"} \\ b = 0.369435675413 \cdot 1.01750971869 \cdot \mathrm{Wd} & \text{"damping ratio equation"} \\ \mathrm{Wn} = 1.4496739182 & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{10}$$

$\boxed{12}$ `// substitute again - mass is given`

$$\begin{bmatrix} k = 2.10155446911 & \text{"spring constant"} \\ b = 0.535561263099 & \text{"damping constant"} \\ \mathrm{Wn} = 1.4496739182 & \text{"natural frequency"} \\ \zeta = 0.184717837706 & \text{"damping ratio"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency"} \end{bmatrix} \tag{11}$$

$\boxed{13}$ `//`