2
```
eq1:=augment(tran([Wn=(sqrt(k/m)),zeta=(b/(2*sqrt(k*m))),Wd=(Wn*sqrt(-zeta^2+1)),zeta=(ln(x0/xn)/(sqrt(4*pi^2+(ln(x0/xn)/n)^2)*n)),Wd=(
frequency equation","damping ratio equation","damped frequency equation","damping ratio from measurements","damped frequency
from measurements"])) // relevant equations
```

$$\begin{bmatrix} \mathrm{Wn} = \sqrt{\frac{k}{m}} & \text{"natural frequency equation"} \\ \zeta = \frac{b}{2\sqrt{km}} & \text{"damping ratio equation"} \\ \mathrm{Wd} = \mathrm{Wn} \cdot \sqrt{-\zeta^2 + 1} & \text{"damped frequency equation"} \\ \zeta = \frac{\ln\left(\frac{x_0}{\mathrm{xn}}\right)}{\sqrt{4\pi^2 + \left(\frac{\ln\left(\frac{x_0}{\mathrm{xn}}\right)}{n}\right)^2} n} & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = \frac{2\pi n}{-t_0 + \mathrm{tn}} & \text{"damped frequency from measurements"} \end{bmatrix} \tag{1}$$

3
```
vars01:=augment(tran([t0=4.15041,x0=0.0702612,tn=17.3807,xn=0.00203278,n=3,m=1]),tran(["start time","starting position","time
at n cycles","position at n cycles","number of cycles","mass"])) // data from curve
```

$$\begin{bmatrix} t_0 = 4.15041 & \text{"start time"} \\ x_0 = 0.0702612 & \text{"starting position"} \\ \mathrm{tn} = 17.3807 & \text{"time at n cycles"} \\ \mathrm{xn} = 0.00203278 & \text{"position at n cycles"} \\ n = 3 & \text{"number of cycles"} \\ m = 1 & \text{"mass"} \end{bmatrix} \tag{2}$$

4
```
eq2:=subst(eq1,vars01) // substitute data into equations
```

$$\begin{bmatrix} \mathrm{Wn} = \sqrt{k} & \text{"natural frequency equation"} \\ \zeta = \frac{b}{2\sqrt{k}} & \text{"damping ratio equation"} \\ \mathrm{Wd} = \mathrm{Wn} \cdot \sqrt{-\zeta^2 + 1} & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{3}$$

5
```
eq3:=rhs_subst(eq2,eq2) // substitute calculated damping ratio and damped frequency into equations
```

$$\begin{bmatrix} \mathrm{Wn} = \sqrt{k} & \text{"natural frequency equation"} \\ \zeta = \frac{b}{2\sqrt{k}} & \text{"damping ratio equation"} \\ \mathrm{Wd} = \sqrt{k} \cdot 0.982791595626 & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{4}$$

6 `eq3[0][0]:=k=solve(eq3[0][0],k)[0] // solve first equation for k`

$$\begin{bmatrix} k = \mathrm{Wn}^2 & \text{"natural frequency equation"} \\ \zeta = \frac{b}{2\sqrt{k}} & \text{"damping ratio equation"} \\ \mathrm{Wd} = \sqrt{k} \cdot 0.982791595626 & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{5}$$

7 `eq4:=normal(rhs_subst(eq3,eq3)) // substitute into equations`

$$\begin{bmatrix} k = \mathrm{Wn}^2 & \text{"natural frequency equation"} \\ \zeta = \frac{b}{2 \cdot \mathrm{Wn}} & \text{"damping ratio equation"} \\ \mathrm{Wd} = 0.982791595626 \cdot \mathrm{Wn} & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{6}$$

8 `eq4[1][0]:=b=solve(eq4[1][0],b)[0] // solve second equation for b`

$$\begin{bmatrix} k = \mathrm{Wn}^2 & \text{"natural frequency equation"} \\ b = 2\zeta \cdot \mathrm{Wn} & \text{"damping ratio equation"} \\ \mathrm{Wd} = 0.982791595626 \cdot \mathrm{Wn} & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{7}$$

9 `eq5:=rhs_subst(eq4,eq4) // substitute into equations`

$$\begin{bmatrix} k = \mathrm{Wn}^2 & \text{"natural frequency equation"} \\ b = 0.369435675413 \cdot \mathrm{Wn} & \text{"damping ratio equation"} \\ \mathrm{Wd} = 0.982791595626 \cdot \mathrm{Wn} & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{8}$$

10 `eq5[2][0]:=Wn=solve(eq5[2][0],Wn)[0] // solve third equation for natural frequency`

$$\begin{bmatrix} k = \mathrm{Wn}^2 & \text{"natural frequency equation"} \\ b = 0.369435675413 \cdot \mathrm{Wn} & \text{"damping ratio equation"} \\ \mathrm{Wn} = 1.01750971869 \cdot \mathrm{Wd} & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{9}$$

11 `eq6:=rhs_subst(eq5,eq5) // substitute into equations`

$$\begin{bmatrix} k = (1.01750971869 \cdot \mathrm{Wd})^2 & \text{"natural frequency equation"} \\ b = 0.369435675413 \cdot 1.01750971869 \cdot \mathrm{Wd} & \text{"damping ratio equation"} \\ \mathrm{Wn} = 1.4496739182 & \text{"damped frequency equation"} \\ \zeta = 0.184717837706 & \text{"damping ratio from measurements"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency from measurements"} \end{bmatrix} \tag{10}$$

12 `eq7:=augment(tran(col(rhs_subst(eq6,eq6),0)),tran(["spring constant","damping constant","natural frequency","damping ratio","damped frequency"])) // substitute again - mass is given`

$$\begin{bmatrix} k = 2.10155446911 & \text{"spring constant"} \\ b = 0.535561263099 & \text{"damping constant"} \\ \mathrm{Wn} = 1.4496739182 & \text{"natural frequency"} \\ \zeta = 0.184717837706 & \text{"damping ratio"} \\ \mathrm{Wd} = 1.42472734321 & \text{"damped frequency"} \end{bmatrix} \tag{11}$$

13