2
```
eq1a:=_lb/_s // target mass flow units
```

$$1\ \mathrm{lb}\,\mathrm{s}^{-1} \tag{1}$$

3
```
eq1:=mdot_c=RPM*disp/2*eta_vol*Pman/(R_air*Tman) // mass flow into cylinder
```

$$\mathrm{mdot\_c} = \frac{\mathrm{Pman} \cdot \mathrm{RPM} \cdot \mathrm{disp} \cdot \eta_{\mathrm{vol}}}{2 R_{\mathrm{air}} \cdot \mathrm{Tman}} \tag{2}$$

4
```
vars01:=tran([RPM=1000/(_mn),disp=185_(inch^3),eta_vol=0.8,Pamb=14.7_lbf/_(inch^2),Tamb=540_Rankine,Pman=13_lbf/_(inch^2),Tman=540_Rankine
// constants with units
```

$$\begin{bmatrix} \mathrm{RPM} = 1000\ \mathrm{mn}^{-1} \\ \mathrm{disp} = 185\ \mathrm{inch}^3 \\ \eta_{\mathrm{vol}} = 0.8 \\ \mathrm{Pamb} = 14.7\ \frac{\mathrm{lbf}}{\mathrm{inch}^2} \\ \mathrm{Tamb} = 540\ {}^\circ\mathrm{R} \\ \mathrm{Pman} = 13\ \frac{\mathrm{lbf}}{\mathrm{inch}^2} \\ \mathrm{Tman} = 540\ {}^\circ\mathrm{R} \\ \mathrm{Vman} = 100\ \mathrm{inch}^3 \\ k = 1.4 \\ R_{\mathrm{air}} = 53.34\ \mathrm{ftlbf}{}^\circ\mathrm{R}^{-1}\mathrm{lb}^{-1} \\ D = 1.6875\ \mathrm{inch} \\ c_t = 0.9 \end{bmatrix} \tag{3}$$

5
```
eq2:=lhs(eq1)=convert(rhs(subst(eq1,vars01)),eq1a) // substitute in values with units
```

$$\mathrm{mdot\_c} = 0.046386948545\ \mathrm{lb}\,\mathrm{s}^{-1} \tag{4}$$

6
```
vars02:=rm_units(vars01) // remove units
```

$$\left[\begin{array}{c} \mathrm{RPM} = 1000 \\ \mathrm{disp} = 185 \\ \eta_{\mathrm{vol}} = 0.8 \\ \mathrm{Pamb} = 14.7 \\ \mathrm{Tamb} = 540 \\ \mathrm{Pman} = 13 \\ \mathrm{Tman} = 540 \\ \mathrm{Vman} = 100 \\ k = 1.4 \\ R_{\mathrm{air}} = 53.34 \\ D = 1.6875 \\ c_t = 0.9 \end{array}\right] \tag{5}$$

```
7 eq3:=subst(eq1,vars02) // substitute values without units
```

$$\mathrm{mdot\_c} = 33.3986029524 \tag{6}$$

```
8 eq4:=multiplier=rhs(rm_units([[eq2]])[0][0])/rhs(eq3) // compute ratio between answers
```

$$\mathrm{multiplier} = 0.00138888888889 \tag{7}$$

```
9 eq5:=lhs(eq1)=normal(evalf(rhs(eq4)*rhs(eq1))) // substitute into original equation
```

$$\mathrm{mdot\_c} = \frac{0.000694444444444 \cdot \mathrm{Pman} \cdot \mathrm{RPM} \cdot \mathrm{disp} \cdot \eta_{\mathrm{vol}}}{R_{\mathrm{air}} \cdot \mathrm{Tman}} \tag{8}$$

```
10 eq6:=tran([Pratio=(Pman/Pamb),flow_c=(sqrt(2*k/(k-1))*sqrt(-Pratio^(k+1/k)+Pratio^(2/k))),mdot_t=(2*c_t*pi/4*D^2*Pamb*sqrt(k/(Tamb*R_ai
// mass flow from manifold
```

$$\left[\begin{array}{c} \mathrm{Pratio} = \frac{\mathrm{Pman}}{\mathrm{Pamb}} \\ \mathrm{flow\_c} = \sqrt{2}\sqrt{\frac{k}{k-1}}\sqrt{-\mathrm{Pratio}^{k+\frac{1}{k}} + \left(\mathrm{Pratio}^{\frac{1}{k}}\right)^2} \\ \mathrm{mdot\_t} = \frac{\pi D^2 \cdot \mathrm{Pamb} \cdot c_t \cdot \mathrm{flow\_c} \cdot \sqrt{\frac{k}{R_{\mathrm{air}} \cdot \mathrm{Tamb}}}}{2} \end{array}\right] \tag{9}$$

```
11 eq6a:=subst(eq6,vars01) // substitute in values with units
```

$$\left[\begin{array}{c} \text{Pratio} = 0.884353741497\,\text{lbflbf}^{-1} \\ \text{flow\_c} = 2.64575131106\sqrt{-\text{Pratio}^{2.11428571429} + \text{Pratio}^{1.42857142857}} \\ \text{mdot\_t} = 1.41371669412 \cdot \text{flow\_c} \cdot \left(14.7\,\frac{\text{lbf}}{\text{inch}^2}\right)\left(2.84765625\,\text{inch}^2\right)\left(0.0069717311682\,{}^\circ\text{R}^{-0.5}\sqrt{{}^\circ\text{R}}\text{ft}^{-0.5}\sqrt{\text{lb}}\text{lbf}^{-0.5}\right) \end{array}\right] \tag{10}$$

```
12 eq6a[0]:=[lhs(eq6a[0][0])=mksa(rhs(eq6a[0][0]))] // simplify units in first equation
```

$$\left[\begin{array}{c} \text{Pratio} = 0.884353741497 \\ \text{flow\_c} = 2.64575131106\sqrt{-\text{Pratio}^{2.11428571429} + \text{Pratio}^{1.42857142857}} \\ \text{mdot\_t} = \text{flow\_c} \cdot \left(0.412579757699\,\text{lbf}{}^\circ\text{R}^{-0.5}\sqrt{{}^\circ\text{R}}\text{ft}^{-0.5}\sqrt{\text{lb}}\text{lbf}^{-0.5}\right) \end{array}\right] \tag{11}$$

```
13 eq6a[2][0]:=lhs(eq6a[2][0])=normal(ufactor(mksa(rhs(eq6a[2][0])),eq1a)) // simplify units in third equation
```

$$\left[\begin{array}{c} \text{Pratio} = 0.884353741497 \\ \text{flow\_c} = 2.64575131106\sqrt{-\text{Pratio}^{2.11428571429} + \text{Pratio}^{1.42857142857}} \\ \text{mdot\_t} = 2.34050505842 \cdot \text{flow\_c} \cdot \left(1\ \text{lbs}^{-1}\right) \end{array}\right] \tag{12}$$

```
14 eq6b:=rhs_subst(rhs_subst(eq6a,eq6a),eq6a) // finish substitutions to find mdot_t
```

$$\left[\begin{array}{c} \text{Pratio} = 0.884353741497 \\ \text{flow\_c} = 0.688943205916 \\ \text{mdot\_t} = 1.61247505841\ \text{lbs}^{-1} \end{array}\right] \tag{13}$$

```
15 eq6d:=subst(eq6,vars02) // substitute values without units
```

$$\left[\begin{array}{c} \text{Pratio} = 0.884353741497 \\ \text{flow\_c} = 2.64575131106\sqrt{-\text{Pratio}^{2.11428571429} + \text{Pratio}^{1.42857142857}} \\ \text{mdot\_t} = 0.412579757699 \cdot \text{flow\_c} \end{array}\right] \tag{14}$$

```
16 eq6e:=rhs_subst(rhs_subst(eq6d,eq6d),eq6d) // finish substitutions to find mdot_t
```

$$\left[\begin{array}{c} \text{Pratio} = 0.884353741497 \\ \text{flow\_c} = 0.688943205916 \\ \text{mdot\_t} = 0.284244020965 \end{array}\right] \tag{15}$$

```
17 eq6f:=multiplier=rhs(eq6b[2][0])[1]/rhs(eq6e[2][0]) // compute ratio between answers
```

$$\text{multiplier} = 5.67285479898 \tag{16}$$

18 `eq6[2][0]:=lhs(eq6[2][0])=normal(evalf(rhs(eq6f)*rhs(eq6[2][0]))) // substitute into third equation`

$$\begin{bmatrix} \text{Pratio} = \frac{\text{Pman}}{\text{Pamb}} \\ \text{flow\_c} = \sqrt{2}\sqrt{\frac{k}{k-1}}\sqrt{-\text{Pratio}^{k+\frac{1}{k}} + \left(\text{Pratio}^{\frac{1}{k}}\right)^2} \\ \text{mdot\_t} = 8.91089948067 D^2 \cdot \text{Pamb} \cdot c_t \cdot \text{flow\_c} \cdot \sqrt{\frac{k}{R_{\text{air}} \cdot \text{Tamb}}} \end{bmatrix} \tag{17}$$

19 `eq7:=Pdot_man=(R_air*Tman/Vman)*(mdot_t-mdot_c) // rate of pressure change equation`

$$\text{Pdot\_man} = \frac{R_{\text{air}} \cdot \text{Tman} \cdot (-\text{mdot\_c} + \text{mdot\_t})}{\text{Vman}} \tag{18}$$

20

22 `eq8:=eqjoin(s1,0) // remove mdot term`

$$\text{Pdot\_man} = \frac{R_{\text{air}} \cdot \text{Tman}}{\text{Vman}} \tag{19}$$

23 `eq8a:=normal(lhs(eq8)=rhs(eq8)*eq1a) // replace with unit mass flow`

$$\text{Pdot\_man} = \frac{R_{\text{air}} \cdot \text{Tman} \cdot \left(1\ \text{lbs}^{-1}\right)}{\text{Vman}} \tag{20}$$

24 `eq9:=subst(eq8a,vars01) // substitute in values with units`

$$\text{Pdot\_man} = 288.036\ \frac{{}^\circ\text{Rftlblbf}{}^\circ\text{R}^{-1}\text{lb}^{-1}\text{s}^{-1}}{\text{inch}^3} \tag{21}$$

25 `eq10:=lhs(eq9)=convert(rhs(eq9),_lbf/_(inch^2)/_s) // simplify units`

$$\text{Pdot\_man} = 3456.432\ \frac{\text{lbfs}^{-1}}{\text{inch}^2} \tag{22}$$

26 `eq11:=subst(eq8,vars02) // substitute in values without units`

$$\text{Pdot\_man} = 288.036 \tag{23}$$

27 `eq12:=multiplier=rhs(eq10)[1]/rhs(eq11) // compute ratio between answers`

$$\text{multiplier} = 12.0 \tag{24}$$

28 `eq13:=lhs(eq7)=rhs(eq12)*rhs(eq7) // substitute into equation`

$$\text{Pdot\_man} = \frac{12.0 R_{\text{air}} \cdot \text{Tman} \cdot (-\text{mdot\_c} + \text{mdot\_t})}{\text{Vman}} \tag{25}$$

29 `eq14:=semi_augment(semi_augment([[eq5]],eq6),[[eq13]]) // add equation to list - final list of equations`

$$\begin{bmatrix} \text{mdot\_c} = \frac{0.000694444444444 \cdot \text{Pman} \cdot \text{RPM} \cdot \text{disp} \cdot \eta_{\text{vol}}}{R_{\text{air}} \cdot \text{Tman}} \\ \text{Pratio} = \frac{\text{Pman}}{\text{Pamb}} \\ \text{flow\_c} = \sqrt{2}\sqrt{\frac{k}{k-1}}\sqrt{-\text{Pratio}^{k+\frac{1}{k}} + \left(\text{Pratio}^{\frac{1}{k}}\right)^2} \\ \text{mdot\_t} = 8.91089948067 D^2 \cdot \text{Pamb} \cdot c_t \cdot \text{flow\_c} \cdot \sqrt{\frac{k}{R_{\text{air}} \cdot \text{Tamb}}} \\ \text{Pdot\_man} = \frac{12.0 R_{\text{air}} \cdot \text{Tman} \cdot (-\text{mdot\_c} + \text{mdot\_t})}{\text{Vman}} \end{bmatrix} \tag{26}$$

30 `lname(eq14)`

$$[\text{mdot\_c}, \text{Pman}, \text{RPM}, \text{disp}, \eta_{\text{vol}}, R_{\text{air}}, \text{Tman}, \text{Pratio}, \text{Pamb}, \text{flow\_c}, k, \text{mdot\_t}, D, c_t, \text{Tamb}, \text{Pdot\_man}, \text{Vman}] \tag{27}$$

31 `eq15:=subst(eq14,vars02) // substitute values without units into all equations`

$$\begin{bmatrix} \text{mdot\_c} = 0.046386948545 \\ \text{Pratio} = 0.884353741497 \\ \text{flow\_c} = 2.64575131106\sqrt{-\text{Pratio}^{2.11428571429} + \text{Pratio}^{1.42857142857}} \\ \text{mdot\_t} = 2.34050505842 \cdot \text{flow\_c} \\ \text{Pdot\_man} = 3456.432\,(-\text{mdot\_c} + \text{mdot\_t}) \end{bmatrix} \tag{28}$$

32 `eq16:=rhs_subst(eq15,eq15) // substitute computed values into other equations`

$$\begin{bmatrix} \text{mdot\_c} = 0.046386948545 \\ \text{Pratio} = 0.884353741497 \\ \text{flow\_c} = 0.688943205916 \\ \text{mdot\_t} = 6.19239432688\sqrt{-\text{Pratio}^{2.11428571429} + \text{Pratio}^{1.42857142857}} \\ \text{Pdot\_man} = 3456.432\,(2.34050505842 \cdot \text{flow\_c} - 0.046386948545) \end{bmatrix} \tag{29}$$

33 `eq17:=rhs_subst(eq16,eq16) // substitute new computed values into other equations to yield initial rate of pressure rise`

$$\begin{bmatrix} \text{mdot\_c} = 0.046386948545 \\ \text{Pratio} = 0.884353741497 \\ \text{flow\_c} = 0.688943205916 \\ \text{mdot\_t} = 1.61247505841 \\ \text{Pdot\_man} = 5413.07705776 \end{bmatrix} \tag{30}$$

$\boxed{34}$